Poisson Access Networks with Shadowing Modelling and Statiscal Inference

Mokhtar Zahdi ALAYA

Supervision : - Dr. Bartłomiej BŁASZCZYSZYN

- Dr. Mohamed Kadhem KARRAY

September 2011

UPMC PARIS UNIVERSITAS

INRIA

Plan

Plan

1. Description of the Model
   - System Model
   - Path-loss factor
   - Interference factor
2. Mathematical Results
   - Analysis of the Path-loss factor
   - Analysis of the Interference factor
   - Path-loss Exponent Estimation

Plan

1. Description of the Model
   - System Model
   - Path-loss factor
   - Interference factor
2. Mathematical Results
   - Analysis of the Path-loss factor
   - Analysis of the Interference factor
   - Path-loss Exponent Estimation

Plan

1. Description of the Model
    - System Model
    - Path-loss factor
    - Interference factor
2. Mathematical Results
    - Analysis of the Path-loss factor
    - Analysis of the Interference factor
    - Path-loss Exponent Estimation

1. Description of the Model

System Model

A well accepted model for the node distribution in wireless networks is the homogeneous Poisson point process (PPP) of intensity λ. Without loss of generality, we can assume that the base stations (BS) are located at the point of a stationary, homogeneous PPP $\Phi := \{X_n, n \in \mathbb{N}\}$ of intensity λ BS km^2 on the plane $\mathbb{R}^2$.

System Model

For a given BS $X \in \Phi$ and give a location $y \in \mathbb{R}^2$ on the plane we denote by $P_X(y)$ the time average, i.e. averaged out over the fading propagation-loss between BS X and the Location y. In what following we will always assume that

$$P_X(y) = \frac{S_X(y)}{l(|X-y|)} \quad (1)$$

$$L_X(y) = \frac{l(|X-y|)}{S_X(y)}. \quad (2)$$

where $l(.)$ is a non-decreasing, deterministic function of the distance between an emitter and a receiver, and $S_X(.)$ is a random shadowing field related to the BS X.

System Model

Regarding the distribution of the marks (shadowing fields) of this process, they are assumed to have the same distribution for all $y \in \mathbb{R}^2$.
For the deterministic path-loss function $l(.)$ the following particular model is often used and will be our default hypothesis in this thesis :

$$\textbf{[H]} \begin{cases} l(r) = (Kr)^{\beta}, \ \textit{where } K > 0 \textit{ and } \beta > 2 \textit{ are some constants.} \\ \beta \textit{ is called the path-loss exponent (PLE),} \\ \textit{For all } y, S_X(y) \textit{ is log-normal random variable variable, } S \textit{ can be} \\ S \overset{D}{=} e^{m+\sigma Z}, \\ \textit{where } Z \textit{ is a standard Gaussian random varaible (with mean 0 a} \end{cases}$$

Path-loss factor

In what follows we will assume that each given location $y \in \mathbb{R}^2$ is served by the BS $X_y^* \in \Phi$ with respect to which it has the highest path-loss $P_{X_y^*}(y)$ (so, in other words, the strongest received signal, given all BS emit with the same power),i.e., such that

$$P_{X_y^*}(y) = \max_{n \in \mathbb{N}} \frac{S_{X_n}(y)}{l(|X_n - y|)}, \tag{3}$$

Path-loss factor

Consequently we have,

$$P_{X_y^*}(y) \geq P_X(y), \; \forall X \; \in \Phi.$$

We notice that $P_X(y)$ is the path-loss experienced by a user located at y with respect to its serving BS. Obviously it determines the quality of the services of this user. In this context we will call *path-loss factor* of a user y and denote by $P_{X_y^*}(y)$. It depends on the location y but also on the path-loss conditions of this location with respect to all BS in the network $P(y) = P(y, \Phi)$. Path-loss factor $P_{X_y^*}(y)$ is typically not enough to determine the qualities of services of a given user.

Interference factor

In wireless networks, interference is one of the central elements in system design, since network performance is often limited by competition of users for common resources. For a given location $y \in \mathbb{R}^2$ the *interference factor* $f(y)$ is defined as

$$f(y) = f(y, \widetilde{\Phi}) = \sum_{X \in \Phi,\ X \neq X_y^*} \frac{P_X(y)}{P_{X_y^*}(y)}, \tag{4}$$

provided X_y^* is well defined. Indeed, $f(y) = \widetilde{f}(y) - 1$ where $\widetilde{f}(y) = \sum_{X \in \Phi} \frac{P(y)}{P_X(y)}$.

Interference factor

Without loss of generality, since the network is homogenous, the interference measure at the origin is representative of the interference seen by all the other receiver nodes in the network is given by,

$$f(o) = \sum_{X \in \Phi,\ X \neq X_o^*} \frac{P_X(o)}{P_{X_o^*}(o)} = \widetilde{f}(o) - 1 = \frac{1}{P_{X^*}} \sum_{X_n \in \Phi} \frac{S_{X_n}}{l(|X_n|)}.$$

Interference factor

The interference power seen by the receiver at the origin can be viewed as a random field or, more specially, as a shot noise process described as

$$I \equiv I(o) := \sum_{n \in \mathbb{N}} \frac{S_n}{l(|X_n|)}. \quad (5)$$

If we define

$$L \equiv L(o) := \min_{n \in \mathbb{N}} L_{X_n}(o), \quad (6)$$

then we can express the interference factor in terms of L and the shot noise I as following

$$f = I \times L - 1. \quad (7)$$

2. Mathematical Results

Invariance of the model with respect to the density of the Shadowing

Taking into account the previous hypothesis **[H]** in chapter 2 that we have assumed to be satisfied by our model. we are going to show that the distribution of the interference factor f does not depend on the intensity λ of the Poisson point process Φ. Let us now construct a new point process $\Phi' = \{Y_n, n \in \mathbb{N}\}$ of intensity 1 by taking $X_n = \frac{Y_n}{\sqrt{\lambda}}$. Indeed using the new expression of P_{X^*}, which is

$$P_{X^*} = \max_n \frac{S_{X_n}}{l(|\frac{Y_n}{\sqrt{\lambda}}|)} = \lambda^{\frac{\beta}{2}} \max_n \frac{S_{X_n}}{l(|Y_n|)}.$$

In that follows, the expression of the interference factor is given by

$$\widetilde{f}(0) = \sum \frac{P_X}{P_{X^*}} = \sum \frac{P_{\frac{Y}{\sqrt{\lambda}}}}{P_{Y^*}}$$

Analysis of the Path-loss factor

Theorem

Consider infinite Poisson process Φ model of the BS, with shadowing whose marginal distribution has finite moment of order $\frac{2}{\beta}$ and for any deterministic path-loss function $0 < l(r) < \infty$. Then, the distribution of P_{X^*} has the following form

$$\mathbb{P}\Big(P_{X^*} \leq r\Big) = \exp\left(-\lambda \int_{\mathbb{R}^2} \Big(1 - F_{S_X}\Big(rl(|X|)\Big)\Big)\, dX\right). \quad (8)$$

Analysis of the Path-loss factor

Remark

Taking into account the hypothesis ***[H]*** of the previous chapter, we are going to show that the distribution function of P_{X^*} depends only on the moment $E[S^{\frac{2}{\beta}}]$ of the shadowing.

Analysis of the Path-loss factor

Example

Assume an infinite Poisson model Φ of BS locations satisfying the hypothesis ***[H]***. Going back to remak (3.2.1) we can get an explicit expression of the probability distribution function of the path-loss, provided $E[S^{\frac{2}{\beta}}] < \infty$, giving as following

$$\mathbb{P}\Big(P_{X^*} \leq r\Big) = \exp\left(- \frac{\lambda\pi}{K^2 r^{\frac{2}{\beta}}} \; e^{\frac{2\sigma^2}{\beta^2} + \frac{2m}{\beta}} \right). \qquad (9)$$

This is the Fréchet distribution with shape $\frac{2}{\beta}$ and scale parameter $\left(\frac{\lambda\pi}{K^2} e^{\frac{2\,r^2}{\beta^2} + \frac{2m}{\beta}}\right)^{\frac{\beta}{2}}$.

Analysis of the Interference factor

Let us consider the point process $\Psi := \{\frac{l(|X_n|)}{S_{X_n}}, X_n \in \Phi\}$, on $\mathbb{R}^+$ as we see Ψ was constructed from the first Poisson point process Φ. The following lemma shows the Poisson criteria of Ψ.

Lemma

Ψ is a non-homogeneous Poisson point process with intensity measure given by

$$\Lambda_\Psi([0,t]) = \frac{\lambda \pi t^{\frac{2}{\beta}}}{K^2} E[S^{\frac{2}{\beta}}]. \qquad (10)$$

Analysis of the Interference factor

Remark

The distribution of any functional of Ψ does not depend on the distribution of the shadowing S but only on the moment $E[S^{\frac{2}{\beta}}]$. we observe in the previous section that the path-loss factor P_X^* and the interference factor $\widetilde{f}$ are some of that functionals.

Analysis of the Interference factor

Now we derive a general expression for the mean interference in networks whose nodes are distributed as a stationary point process $\Phi = \{X_1, X_2, ...\} \subset \mathbb{R}^2$ of intensity λ.

Proposition

In the Poisson network with deterministic path-loss function, the distribution of the interference factor $f(0)$ does not depend on the marginal distribution of shadowing field $S_X(.)$ provided $E[S^{\frac{2}{\beta}}] < \infty$. Moreover, we have $E[f(0)] = \frac{2}{\beta - 2}$.

Analysis of the Interference factor

Now we derive a general expression for the mean interference in networks whose nodes are distributed as a stationary point process $\Phi = \{X_1, X_2, ...\} \subset \mathbb{R}^2$ of intensity λ.

Remark

we have seen that from the first section of this chapter that the interference factor f does not depend on the intensity measure λ of the Poisson point process Φ. Now using this observation and taking into account $E[S^{\frac{2}{\beta}}] < \infty$ yield that the distribution of interference factor also does not depend not only on the distribution of the shadowing but also on the moment $E[S^{\frac{2}{\beta}}]$ of the shadowing. to see this from remark (), the intensity measure of the process Ψ is $\Lambda_\Psi([0,t]) = \frac{\lambda \pi t^{\frac{2}{\beta}}}{K^2} E[S^{\frac{2}{\beta}}]$, we can consider a new intensity $\lambda' = \frac{\lambda}{E[S^{\frac{2}{\beta}}]}$. Henceforth, the new intensity measure is $\Lambda'_{\ }([0, t]) = \frac{\lambda\pi}{\ }$

Distribution function of the interference factor f

T

he Laplace transform of the shot noise $I = \sum_{n \in N} \frac{S_n}{l(|X_n|)}$ is given by

$$\mathcal{L}_I = exp\Big(-2\pi\lambda \int_0^{\infty} \Big(1 - \mathcal{L}_S(\frac{t}{l(r)}\Big) r dr \Big),$$

where $\mathcal{L}_S(t) = E[e^{-t\,S}]$.

Distribution function of the interference factor f

Corollary

The Laplace functional $\mathcal{L}_I(t)$ of the shot noise I verifies,

$$\mathcal{L}_I(t) = exp\Big(- \frac{2\pi\lambda t^{\frac{2}{\beta}}}{\beta K^2} \Gamma(-\frac{2}{\beta}) E[S^{\frac{2}{\beta}}] \Big). \quad (11)$$

Distribution function of the interference factor f

Proposition (Karray 2011)

The Laplace functional of the interference factor f is given by

$$E[e^{-zf}] = \frac{1}{e^{-z} + z^{\frac{2}{\beta}}\left(\Gamma(1-\frac{2}{\beta}) - \Gamma(1-\frac{2}{\beta}, z)\right)}. \tag{12}$$

where $\Gamma(a) = \int_0^\infty t^{a-1}e^{-a}dt$ is the gamma function and $\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-a}dt$ is the upper incomplete gamma function.

Joint distribution path-loss interference factors

(Karray 2011)

The joint distribution of the path-loss interference factor is given by

$$E\Big[\mathbf{1}\{P_{X^*} \leq u\}\, e^{-zI}\Big] = exp\Big(-\frac{2\pi\lambda}{\beta\, K^2} E[S^{\frac{2}{\beta}}]\, z^{\frac{2}{\beta}} \Big[\Gamma(-\frac{2}{\beta})+\Gamma(-\frac{2}{\beta}, u\, z)\Big]\Big). \tag{13}$$

Corollary

The path-loss and the interference factors are not independent random variables.

Path-loss Exponent Estimation

In wireless channels, the path loss exponent (PLE) has a strong impact on the quality of the links, and hence, it needs to be accurately estimated for the efficient design and operation of wireless networks. Consider our model with the hypothesis ***[H]***, from theorem equation (3.2) we have the probability distribution function of the path-loss factor P_{X^*} as following.

$$\mathbb{P}\Big(P_{X^*} \leq r\Big) = \exp\left(-\frac{\lambda\pi}{K^2 r^{\frac{2}{\beta}}}\ e^{\frac{2\sigma^2}{\beta^2}+\frac{2m}{\beta}} \right).$$

Path-loss Exponent Estimation

$$log\Big(- log\Big[\mathbb{P}\Big(P_{X^*} \leq e^t\Big)\Big]\Big) = A\,t + B \text{ where,}$$

$$\begin{cases} A = -\frac{2}{\beta} \\ B = \Big[log(\frac{\lambda\pi}{K^2}) + \Big(\frac{2\sigma^2}{\beta^2} + \frac{2m}{\beta}\Big)\Big]. \end{cases}$$

References

- Błaszczyszyn, B. and Karray, M. (2011) :
  "How the path-loss with log-normal shadowing impacts the quality of service in cellular networks ; why blocking probability is not always increasing in the shadowing variance",
  Tech. Rep. HAL-INRIA.

- Błaszczyszyn, B., Karray, M. K. and Klepper, F.-X. (2010) :
  "Impact of the geometry, path-loss exponent and random shadowing on the mean interference factor in wireless cellular networks",
  In Proc. of IFIP WMNC. Budapest, Hungary.

References

- Haenggi, M., Andrews, J. G., Baccelli, F., Dousse, O. and Franceschetti, M. (2009) :
"Stochastic Geometry and Random Graphs for the Analysis and Design of Wireless Networks",
IEEE Journal on Selected Areas in Communications, vol. 27.

- El Karoui N., C. Kapoudjian, E. Pardoux, S. Peng et M.C. Quenez (1997) :
"Reflected Solutions of Backward SDE and Related Obstacle Problems for PDEs",
Ann. Probab., 25, 2, 702-737.

Thank you.